Modeling Neural Spiking with Point Processes Nonparametrically: A Convex Optimization Approach

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Characterizing neural spiking activity as a function of environmental stimuli, and intrinsic effects such as a neuron’s own spiking history and concurrent ensemble activity is important in neuroscience. Such a characterization is complex and there is increasing need for a broad class of models to capture such details. Point process models have been shown to be very useful in characterizing neural spiking activity \cite{1}. The likelihood of a point process \( \{N(t)\}_{t=0}^{T} \) is completely defined by its conditional intensity function

\[
\lambda(t|x_t) \equiv \lim_{\Delta \to 0} \frac{P(N(t + \Delta) - N(t) = 1|x_t)}{\Delta}
\]

where \( x_t \) corresponds to previous spiking activity, \( \{N(\tau)\}_{\tau=0}^{t} \), as well as any latent environmental stimuli. Most point process models are parametric as they are often efficiently computable, the parameters may be related back to physiological and/or environmental factors, and they have nice asymptotic properties when \( \lambda(t|x_t) \) lies in the assumed parametric class \cite{1}. However, if \( \lambda(t|x_t) \) does not lie in the assumed class, misleading inferences can arise. Nonparametric methods are attractive due to fewer assumptions, but very few efficient methods for estimating \( \lambda(t|x_t) \) are known. We propose a computationally efficient method for nonparametric maximum likelihood estimation when \( \lambda(t|x_t) \) is assumed to be Lipschitz continuous \cite{2}.

We are given neural spiking activity observations \( \{N_i\}_{i=1}^{M} \) once every \( \Delta = 1 \) ms that result from \( \{x_i\}_{i=1}^{M} \), known stimuli and the neuron’s own spiking history. \( \hat{\lambda}_i \) is the estimate of \( \lambda(t|x_t) \) at millisecond \( i \). We minimize the negative log likelihood of the point process subject to the Lipschitz continuity constraints:

\[
\min_{\lambda} \sum_{i=1}^{M} -N_i \log(\hat{\lambda}_i) + \hat{\lambda}_i \Delta \\
\text{s.t.} \quad \left| \log(\hat{\lambda}_i) - \log(\hat{\lambda}_j) \right| \leq K \|X_i - X_j\|_{\infty}, \quad i < j, \quad j = 1, \ldots, M \tag{1a}
\]

We show that (1a) is convex, as are the constraints in (1b); so (1) is a convex optimization problem and thus efficiently solvable. We develop an equivalent problem with separable \cite[Sec. 3]{3} structure in (1a) and linear structure in (1b) to represent its dual in closed form. Thus (1) can be solved using very efficient unconstrained methods, such as gradient descent. We apply our method to goldfish retinal ganglion neural data and compare results to inhomogeneous Poisson and inverse Gaussian parametric models. We assess goodness-of-fit via the time-rescaling theorem and measure model uncertainty via bootstrapping \cite{4}.

References


